The St. Petersburg Paradox

"Everything that happens once can never happen again. But everything that happens twice will surely happen a third time."
Paulo Coelho (1947 -), Brazilian novelist

Daniel Bernoulli, the famed 18th century Swiss mathematician known for his pioneering work on probability theory, was living in St. Petersburg around 1738 when he stumbled upon a curious anomaly in game theory[1]. Whereas the "expected value" of a random variable with a finite number of outcomes is the probability weighted average of those outcomes[2], he noticed that when the expected value approached "infinity", people began to behave somewhat "irrationally". This is best illustrated with an example where the participant in a game of chance is offered to toss a coin in which an initial payoff of $2 is doubled if the outcome is heads and the game terminates if the result is tails. The mathematically derived expected value in this instance happens to be infinite, because there is a chance, even if it is slim, that the payoff ends up being infinitely large[3]. This would happen if heads were to turn up at every single flip of the coin which, of course, would also mean that the game never ends.

The paradox lies in the divergence between the "expected" and "actual" behavior of participants when presented with such an opportunity. A rational gambler should theoretically be willing to pay just about any entry fee to participate in a game where the return could potentially be infinitely large, because any such fee will always be less than the "expected payoff" of the game! Thus, the gambler should be willing to play no matter how large the entry fee, but that is not what has been observed: participants will have limits beyond which they won't participate[4]. This paradox is not all that innocuous if we consider that the probabilistic theories from which it arises have guided economic thinking for over three centuries!

At the core of the problem is the assumption that the participant's actions are "irrational" because they do not agree with the model, when it is the model itself that we should be questioning. This point has been most recently illustrated by Ole Peters, a theoretical physicist in the UK, who has been causing quite a stir amongst economists[5]. His premise is that certain outliers in samples can sometimes create such distortions in the outcomes that the model fails to correctly predict behavior, which is then wrongly labeled as being "irrational" when in fact it is totally "rational".

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[1] https://en.wikipedia.org/wiki/Game_theory
[2] To illustrate this, if you, for example, have a 50% chance of winning $100 and 50% chance of losing $100, your expected outcome is going to be exactly $0 (0.5x100 + 0.5x-100).
[3] https://en.wikipedia.org/wiki/St._Petersburg_paradox
[4] https://plato.stanford.edu/archives/fall2004/entries/paradox-stpetersburg/
[5] https://www.bloomberg.com/news/articles/2020-12-11/everything-we-ve-learned-about-modern-economic-theory-is-wrong

In Peters' oft quoted example, if you are given $100 and that amount increases by 50% when the coin lands on heads and loses 40% if the result is tails, assuming an equal probability of obtaining heads or tails, you should always be willing to play the game just because your potential payoff (50% increase) should on average be greater than your potential loss (40% decrease)[6]. Most people will decline the offer, however, and rightly so, because when you play the game enough times, you almost always end up with less than the initial $100, if the probability of obtaining heads or tails is exactly 50%!

Even more intriguing is what happens when you run the experiment through a "Monte Carlo simulation"[7] that calculates the outcome of literally thousands of participants, all flipping coins simultaneously very many times. What comes out of this is that after several hundred iterations, a diminishing percentage of participants amass a disproportionately large fortune. This is the point at which the model appears to "break down", as the mean expected return of the entire population of participants (the average of all outcomes) becomes skewed well above the median expected return (the halfway point return value of total participants).

It basically means that what most of the participants are likely to gain will be nowhere near the mean expected return, they will most likely end up with a figure well below the initial $100[8]. In other words, the rational thing to do is stay clear, but because the "outcome" suggests otherwise, this decision is labeled as irrational. Peters' premise might be nothing more than just a curiosity at this stage, creating the occasional chatter on the blogosphere, drawing the attention of a couple of luminaries[9].

It does, however, contain the potential seeds of disruption that promises to upend an entire industry, from risk management to the pricing of assets, not to mention the behavioral economics currently being used to fight the pandemic. There is still some way to go before anything of significance is validated, but just like the process of climate change or the COVID-19 epidemic before it became a global pandemic, it does help to pay attention to the ripples before they turn into destructive waves. Bernoulli was definitely onto something!

Where Do We Go From Here?

Markets began the year on a strong footing, boosted by sentiment that has turned increasingly confident that a rapid pickup in economic activity is likely to happen later in the year. The confidence is being fueled by four main developments in recent months, two of which are going to reinforce the tailwinds of the economy (Biden leadership, economic stimulus), whilst the others help attenuate the pandemic driven headwinds (vaccinations, adaptation). The first few weeks and months of the Biden presidency should lead to a significant diminishment in uncertainty on a broad range of geopolitical matters, contributing to an improvement in the global economic recovery, especially as trade tensions that have been brewing over the last four years subside.

The massive quantities of liquidity that have been injected into economies across the globe are also likely to be maintained indefinitely, as central banks keep the stimulus taps wide open in response to the ongoing pandemic-driven drawdowns. The Treasury has already printed a record quantity of money in 2020, enough to make up for most of the economic losses from COVID-19[10] and is likely to continue doing so as long as the pandemic imposed lockdowns create further strain on the economy. The Fed is also likely to remain fully accommodative, which means that tapering won't be

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[6] Each outcome weighted by its probability, 0.5*$50 + 0.5*-$40) is $5 or 5% of your wealth.
[7] https://en.wikipedia.org/wiki/Monte_Carlo_method
[8] https://jasoncollins.blog/2020/01/22/ergodicity-economics-a-primer/
[9] Michael Mauboussin, the former head of global financial strategies at Credit Suisse, Rick Bookstaber, who helped draft the Volcker Rule, Emanuel Derman, a pioneer of quantitative investing, and Nassim Taleb the famed author of the Black Swan have all supported Peters' research on ergodicity economics.
[10] The U.S. alone pumped $3.3 trillion of stimulus into its economy, a large chunk of it in terms of pay checks following the massive surge in COVID driven layoffs, bringing public debt levels to above 100% of GDP.

happening any time soon, unless there is a sharp and sustained rebound in the economy that starts pushing prices higher. The ECB and other central banks that are also facing structurally weakened economies of their own will also be following a similar path, conducting further stimulus measures to counter the lingering effects of lockdowns.

The increased production and rollout in vaccines across the globe should help accelerate the process of achieving "herd immunity" levels after which we can expect a significant pickup in activity, as confinement measures are gradually relaxed for good. Once the signs that the pandemic is retreating are confirmed, consumption demand will likely experience a sharp surge, as households and businesses scramble to make up for "lost time", before returning back to their pre-COVID levels. The supply side response is going to involve a brisk ramp up in production, combined with a temporary increase in prices as they struggle to meet demand, which should help boost profit margins.

Multiples that have risen well above historical averages should begin to shrink over time, which should pave the way for further gains in the markets in coming months. The big uncertainty now is how rapidly the virus can be "tamed", because this will determine not only when we can expect the rebound to take place, but also how vigorous it will likely be. The longer the semi-confinement measures remain in force, the more significant is likely to be their damaging effects on the economy[11]. But then we have adapted to harsher environments; treating the sick and continuing to operate despite multiple constraints will make each successive wave more tolerable than the previous one.

The upbeat sentiment may be warranted when we consider all four developments in combination, but there are still plenty of reasons to be cautious. As described in the recently published OXFAM report[12], the COVID pandemic has brought to attention global wealth inequalities, which will continue to fuel toxic populism, and emerging new variants of the virus could still prolong the pandemic. China and the U.S. are at the early stages of a technology arms race collision course, climate change is upending the world in uncertain ways[13] and the tools that we have been relying on to steer economies and manage markets may be flawed. So yes, the skies have not cleared yet, but there is enough out there to be cautiously optimistic about the future.

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[11] Economic recessions that are triggered by natural disaster type events tend to recuperate over a much shorter period than those that occur as a result of a financial crisis, because the structural damage from the latter causes is usually more significant.
[12] https://www.oxfam.org/en/press-releases/mega-rich-recoup-covid-losses-record-time-yet-billions-will-live-poverty-least
[13] https://www.nytimes.com/2021/01/25/opinion/new-normal-climate-catastrophes.html?referrer=masthead